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A MATHEMATICAL MODEL FOR INSECT POPULATION IN LIMITED RESOURCES

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ABSTRACT

Insect are the main problem for our environment under limited resources. For agriculture and food industries their growth of population is very fetal for crops. Among all the most conspicuous insect Lepidopetra (Moths) is only done by larva in the green houses and the fields. In this paper, a mathematical model for controlling the population of insect (moths) who damaged the crops are discussed.

KEYWORDS: Population, Insect, Moths, Mortality, Density, Toxic Substances

The major human diseases are produced by microorganism conveyed by insect, which serve as vectors of pathogens. Insects are responsible for two major kinds of damage to growing crops. First is direct injury done to the plant by feeding insect, which eats leaves in stems, fruit, or roots. There are more than hundred of pest species of this type, both in larvae and adults. Various attempt have been made to develop the mathematical models of their population dynamics by (Barlow, 1982). There are different types of insect species are declining across most taxonomic lineages in most regions of world, and at potentially increasing rates as anthropogenic impacts intensify globally (Bell *et al.*, 2019). Moth species associated with grass or herb host plants were more severely affected, as were ground beetle species that were closely associated with xerophytic habitats (Didham *et al.*, 2020). The population of insect has a birth rate and death rate. The first and simplest law of population by (Malthus, 1978)

$$\frac{dP}{dt} = aP$$

Which leads the exponential growth equation is $P(t) = P_0 e^{at}$

This model is valid only in limited resources

Where $p(t)$ denotes the population at time t and $a > 0$ is the specific growth rate.and P_0 is the population at some arbitrary time

$t = 0$. According to (Perl, 1925) developed another model for lmiter resources

$\frac{dP(t)}{dt} = aP(t) - bP(t)^2$, t>0

There are many models for growth by (Kapoor, 1985) assuming different law of by (Shehata and Marr, 1971). Moths can play as biodiversity indicator of the environmental impacts of human activity (Dennis *et al.*, 2019). The insect moths produced toxic substances by bacteria become a limiting factor to their further growth by increasing rapidly the mortality rate of organism because mouths spread saliva on the leaf preventing further sucking and movement on the individuals so causing death by eating toxic substance. Aphid populations show periodic fluctuations and many causes are attributed to their dynamic (Braec *et al.*, 2014). To elucidate this we have developed the following model.

FORMULATION OF THE MODEL AND ITS SOLUTION

Let $P(t)$ is the total number individuals at time t.Ammusing the mortality rate is affected by two factors one is increasing density of insects population and secondly is affected by increasing concentration of toxic product produced by insect, the insect population growth considered as

$$\frac{dP(t)}{dt} = aP(t)\left(1 - \frac{P(t)}{K}\right) - KrP^2(t)$$

Where K is a positive constant and r denotes the concentration of toxic product.

$$\frac{dP(t)}{dt} = aP(t)\left\{\left(1 - \frac{P(t)}{K}\right) - \frac{Kr}{a}P(t)\right\} \quad \ldots(1)$$

or $\quad adt = \frac{1}{P(t)\left\{\left(1-\frac{P(t)}{K}\right)-\frac{Kr}{a}P(t)\right\}}dP$

or $\quad adt = \frac{KdP(t)}{P(t)\left\{K-P(t)(1+\frac{k'r}{a}\right\}}$ where $K' = K^2 > 0$

or $\quad adt = \frac{1}{P(t)} + \frac{M}{\{K-MP(t)\}}$, where $M = \left(1 + \frac{K'r}{a}\right)$ a constant

Now integrating both side we have,

or $\quad at = \int_{P_{0(t)}}^{P(t)} \left\{\frac{1}{P(t)} + \frac{M}{\{K-MP(t\}}\right\} dP(t)$

or $\quad at = [\log P(t) - \log\{K - MP(t\}]_{P_0(t)}^{P(t)}$

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or $\quad at = \log\left[\frac{P(t)}{\{K-MP(t\}}\right] - \log\left[\frac{P_0(t)}{K-MP_0(t)}\right]$

or $\quad at = \log\frac{P(t)\{K-MP_0(t)\}}{P_0(t)\{K-MP(t\}}$

or $\quad e^{at} = \frac{P(t)\{K-MP_0(t)\}}{P_0(t)\{K-MP(t\}}$

or $\quad e^{at}P_0(t)\{K-MP(t\} = P(t)\{K-MP_0(t)\}$

or $\quad P(t) = \frac{P_0(t)K}{(K-MP_0(t)e^{-at}+MP_0(t))} \quad \ldots(2)$

RESULTS AND DISCUSSION

Equation (2) represent the size of the insect populations i.e.,the density of the population at time t. obviously as $t \to \infty, P(t) \to K$.

So from equation (1)

$$\frac{d^2P(t)}{dt^2} = \frac{a}{K^2}P(t)\{K-2K'P(t)\}\{K-K''P(t)\}$$

Where $K' = \left(1-\frac{K^2r}{2a}\right)$ and $K'' = \left(1+\frac{K^2r}{a^2}\right)$

If $(K-2K'P(t)) > 0$ i.e., $\{K-K''P(t)\} > P(t) > 0$ then $\frac{d^2P(t)}{dt^2} > 0$

So that the rate of increase of $\frac{dp(t)}{dt}$ increases with time. This shows that there is an accelerated growth of the insect population in the range $0 < p(t) < \frac{K}{2K'}$. If $\frac{K}{2K'} < P(t) < \frac{K}{K''}$ then

$\{K' - 2K'P(t)\} < 0$ and $\{K-K''P(t)\} > 0$, so that $\frac{d^2P(t)}{dt^2} < 0$ that is the rate of $\frac{dp(t)}{dt}$ decreases with time thus there is a retarded growth of insect population in the range $\frac{K}{2K'} < P(t) < \frac{K}{K''}$ where $K' = K'' \cong 1$ i.e.the population in the range $\frac{K}{2} < P(t) < K$.

CONCLUSION

In above model we have observed that the insect populations fluctuate between two range. These values occur when certain population reach s sufficiently high density that is insect population initially increases but gradually the population decreases and population tends to zero that is population tends to extinction.

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REFERENCES

Barlow N.D., 1982. Modelling aphids population, New Zeland. J. Ecol., **4**:52-55.

Bell J.R., Botham M.S., Henrys P.A., Leech D.I., Pearce□Higgins J.W., Shortall C.R., Brereton T.M., Pickup J. and Thackeray S.J., 2019. Spatial and habitat variation in aphid, butterfly, moth and bird phenologies over the last half century. Global Change Biology, **25**: 1982-1994.

Braec M., Honek A., Pekar S. and Martinkova Z., 2014. Population dynamics of Aphids on cereals Digging in the time-series data to reveal population regulation caused by temperature journal of PLOS Sep 3 (2014), https://doi.org/10.1371/journal.pone.0106228.

Dennis E.B., Brereton T.M., Morgan B.J.T., Fox R., Shortall C.R., Prescott T. and Foster S., 2019. Trends and indicators for quantifying moth abundance and occupancy in Scotland. Journal of Insect Conservation.

Didham R.K., Basset Y., Collins C.M., Leather S.R., Littlewood N.A., Menz M.H.M., Müller J., Packer L., Saunders M.E., Schönrogge K., Stewart A.J.A., Yanoviak S.P. and Hassall C., 2020. Interpreting; insect declines: seven challenges and a way forward. Insect Conservation and Diversity, **13**: 103–114.

Kapoor J.N., 1985. Mathematical Models in Biology and Medicine, affiliated East-Wet press, New Delhi.

Malthus T.R., 1978. An essay on the principles of population, st. Paul, s London.

Pearl R., 1925. The Biology of population growth, Knopf, New York.

Shehata T.E. and Marr A.G., 1971. Effect of nutrient concentration on the growth of Escherichia coli. J. Bacteriol, **107**: 210-16.